SIDDHARTH GROUP OF INSTITUTIONS:: PUTTUR (AUTONOMOUS)
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QUESTION BANK (DESCRIPTIVE)

Subject with Code: Mathematical Foundation of Computer Science (25MC9101) **Course & Branch**: MCA
Year & Sem: I-MCA & I-Sem **Regulation:** R25

UNIT –I

THE FOUNDATIONS LOGIC AND PROOFS

1	**a)**	Explain the connectives and their truth tables.	**[L2][CO1]**	**[6M]**
	b)	Construct the truth table for the following formula $(P \wedge \neg Q) \rightarrow R.$	**[L3][CO1]**	**[6M]**
2	**a)**	Construct the truth table for the following formula $\neg(\neg p \vee \neg q)$	**[L3][CO1]**	**[6M]**
	b)	Define converse, inverse contra positive with an example.	**[L1][CO1]**	**[6M]**
3	**a)**	Define NAND, NOR & XOR and give their truth tables.	**[L1][CO1]**	**[6M]**
	b)	Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent by using truth tables.	**[L2][CO1]**	**[6M]**
4	**a)**	Prove that $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology.	**[L3][CO1]**	**[6M]**
	b)	Show that the value of $(P \rightarrow Q) \wedge (P \rightarrow R)$ is logically equivalent to $P \rightarrow (Q \wedge R)$.	**[L2][CO1]**	**[6M]**
5	**a)**	Define Predicates, simple and compound statement function with example.	**[L1][CO1]**	**[6M]**
	b)	Define Quantifiers and types of Quantifiers with examples.	**[L1][CO1]**	**[6M]**
6	**a)**	Verify the validity of the following arguments: Lions are dangerous animals, There are lions. Therefore, there are dangerous animals.	**[L3][CO1]**	**[6M]**
	b)	Show that $(\exists x)\ M(x)$ follows logically from the premises $(\forall x)(H(x) \rightarrow M(x))\ and\ (\exists x)H(x)$	**[L2][CO1]**	**[6M]**
7	**a)**	Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$	**[L3][CO1]**	**[6M]**
	b)	Explain about Nested Quantifiers with example.	**[L2][CO1]**	**[6M]**
8	**a)**	Prove that $(\forall x)(P(x) \rightarrow (Q(y) \wedge R(x))), (\exists x)P(x) \Rightarrow Q(y) \wedge (\exists x)(P(x) \wedge R(x))$	**[L3][CO1]**	**[6M]**
	b)	Prove that $(n + 1)^3 \geq 3^n$ if n is a positive integer with $n \leq 4$. By using method of exhaustion	**[L2][CO1]**	**[6M]**
9		Use indirect method of proof to show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$	**[L2][CO1]**	**[12M]**
10	**a)**	Prove that if n is an integer, then $n^2 \geq n$. by proof by cases method	**[L2][CO1]**	**[6M]**
	b)	Define Forward and Backward Reasoning with examples	**[L1][CO1]**	**[6M]**

UNIT –II

BASIC STRUCTURES, SETS, FUNCTIONS, SEQUENCES, SUMS, MATRICES AND RELATIONS

1	**a)**	Define (i) Equal sets (ii) Empty set (iii) Subset (iv) The size of a set (v) Power set (vi) Cartesian Product of two sets	**[L1][CO2]**	**[6M]**
	b)	Use set builder notation and logical equivalences to establish (i) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ (ii) $\overline{A \cup B} = \overline{A} \cap \overline{B}$	**[L2][CO2]**	**[6M]**
2	**a)**	Define a function and write the types of functions	**[L1][CO2]**	**[6M]**
	b)	If f: R → R such that $f(x) = 2x + 1\ and\ \ g : R \to R\ such\ that\ \ g(x) = \frac{x}{3}$ then verify that $(gof)^{-1} = f^{-1}og^{-1}$.	**[L3][CO2]**	**[6M]**
3	**a)**	Let $f(x) = x + 3, g(x) = x - 4\, and\, h(x) = 5x$ are functions from $R \to R$ where R is the set of real numbers. Find $f \circ (g \circ h)\ and\ (f \circ g) \circ h$.	**[L3][CO2]**	**[6M]**
	b)	Find the inverse of the following functions: $(i) f(x) = \frac{10}{\sqrt[5]{7-3x}}\ (ii) f(x) = 4e^{(6x+2)}$	**[L3][CO2]**	**[6M]**
4	**a)**	Find (i) The Fibonacci numbers f_2, f_3, f_4, f_5, and f_6. (ii) Formulae for the sequences with the following first five terms: (a) 1, 1/2, 1/4, 1/8, 1/16 (b) 1, 3, 5, 7, 9 (c) 1, −1, 1, −1, 1.	**[L3][CO2]**	**[6M]**
	b)	If a and r are real numbers and $r \neq 0$, then show that $\sum_{j=0}^{n} ar^{j} = \begin{cases} \frac{ar^{n+1} - a}{r - 1} & if\ r \neq 1 \\ (n+1)a & if\ r = 1 \end{cases}$	**[L3][CO2]**	**[6M]**
5	**a)**	Show that if A and B are countable sets, then $A \cup B$ is also countable.	**[L2][CO2]**	**[6M]**
	b)	Find the Boolean product AB and BA, where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$	**[L3][CO2]**	**[6M]**
6		Let $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$. Then find A^4 by using Boolean product	**[L3][CO2]**	**[12M]**
7	**a)**	Define Relation? Write the properties of relations.	**[L1][CO2]**	**[6M]**
	b)	Let A={0,1,2,3,4}.Show that the relation $R = \{(0,0),(0,4),(1,1),(1,3),(2,2),(3,1),(3,3),(4,0),(4,4)\}$ is an equivalence relation.	**[L3][CO2]**	**[6M]**
8	**a)**	Define an equivalence relation? If R be a relation in the set of integers Z defined by $R = \{(x, y) : x \in Z, y \in Z, (x - y)\, is\ divisible\ by\ 6\}$. Then prove that R is an equivalence relation.	**[L1][CO2]**	**[6M]**
	b)	Let A = { 1,2,3,4} and let R be the relation on A defined by xRy if and only if “x divides y”, written x/y. Then i) Write down R as a set of ordered pairs. ii) Draw the diagraph of R.	**[L2][CO2]**	**[6M]**
9		Define transitive closures. Let A = { 1,2,3}and R= {(1,2),(2,3),(3,1)}.Find the reflexive, symmetric and transitive closures of R, using composition of matrix relation of R.	**[L1][CO2]**	**[12M]**
10		Let A be a given finite set and P(A) its power set. Let $\subseteq$ be the inclusion relation on the elements of P(A). Draw the Hasse diagram of (P(A) , $\subseteq$) for (i) A = { a } (ii) A ={ a ,b} (iii) A = { a,b,c } (iv) A = { a,b,c,d }.	**[L2][CO2]**	**[12M]**

UNIT –III

ALGORITHMS, INDUCTION AND RECURSION

1	**a)**	Write the properties of algorithms	**[L1][CO3]**	**[6M]**
	b)	Define Linear search algorithm and Binary search algorithm	**[L1][CO3]**	**[6M]**
2	**a)**	Explain Bubble sort and Insertion sort.	**[L3][CO3]**	**[6M]**
	b)	Use the bubble sort to put 3, 2, 4, 1, 5 into increasing order	**[L2][CO3]**	**[6M]**
3	**a)**	Use the insertion sort to put the elements of the list 3, 2, 4, 1, 5 in increasing order.	**[L2][CO3]**	**[6M]**
	b)	Define Big-O Notation with an example	**[L1][CO3]**	**[6M]**
4	**a)**	Let $\mathbf{f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}$, where $\mathbf{a_0, a_1, \ldots, a_{n-1}, a_n}$ are real numbers. Then show that f(x) is $\mathbf{O(x^n)}$.	**[L3][CO3]**	**[6M]**
	b)	Define Big-Omega and Big-Theta Notation with an example	**[L1][CO3]**	**[6M]**
5	**a)**	Define Time Complexity.	**[L1][CO3]**	**[2M]**
	b)	State and prove The Halting Problem.	**[L3][CO3]**	**[10M]**
6	**a)**	Show that if n is a positive integer, then $\boldsymbol{1 + 2 + 3 + --- + n = \frac{n(n+1)}{2}}$ by using mathematical induction method	**[L2][CO4]**	**[6M]**
	b)	Use mathematical induction to show that $\mathbf{1 + 2 + 2^2 + --- + 2^n = 2^{n+1} - 1}$	**[L2][CO4]**	**[6M]**
7	**a)**	Explain mathematical induction and strong induction.	**[L3][CO4]**	**[6M]**
	b)	Show that if n is an integer greater than 1, then n can be written as the product of primes. By using strong induction method	**[L2][CO4]**	**[6M]**
8	**a)**	State and prove division algorithm. By using well-ordering property	**[L1][CO4]**	**[6M]**
	b)	Define Recursively defined function, Suppose that f is defined recursively by f(0)=3, f(n+1)=2f(n)+3, then find f(1), f(2), f(3) and f(4).	**[L1][CO4]**	**[6M]**
9	**a)**	Show that when ever $\boldsymbol{n \geq 3}$, $\boldsymbol{f_n > \alpha^{n-2}}$, where $\boldsymbol{\alpha = \frac{(1+\sqrt{5})}{2}}$	**[L2][CO4]**	**[6M]**
	b)	Define set of rooted trees, extended binary trees, full binary trees recursively	**[L1][CO4]**	**[6M]**
10	**a)**	If T is a full binary tree, then show that $\boldsymbol{n(T) \leq 2^{h(T)+1} - 1}$ by using structural induction	**[L3][CO4]**	**[6M]**
	b)	Define Recursive algorithm. Give a recursive algorithm for computing n!, where n is a non-negative integer.	**[L1][CO4]**	**[6M]**

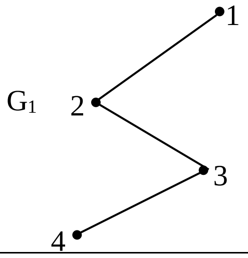
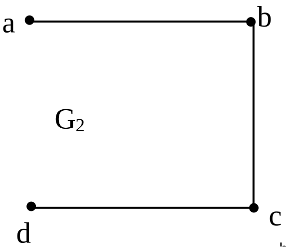
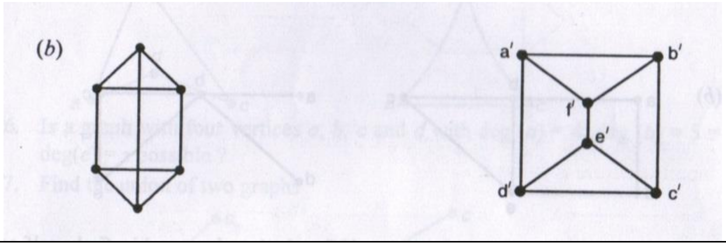
UNIT –IV

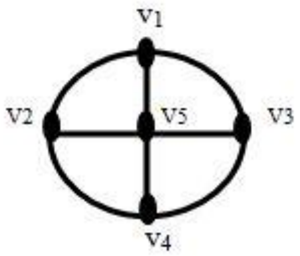
DISCRETE PROBABILITY AND ADVANCED COUNTING TECHNIQUES

<table>
<tr><td>1</td><td>a)</td><td>A class consists of 6 girls and 10 boys. If a committee of 3 is chosen at random from the class, find the Probability that (i) 3 boys are selected (ii) Exactly 2 girls are selected.</td><td>[L1][CO5]</td><td>[6M]</td></tr>
<tr><td></td><td>b)</td><td>If three coins are tossed, Find the probability of getting
i) 3 heads ii) 2 heads iii) no heads.</td><td>[L1][CO5]</td><td>[6M]</td></tr>
<tr><td>2</td><td>a)</td><td>Two cards are selected at random from 10 cards numbered 1 to 10. Find the probability that the sum is even if (i) The two cards are drawn together. (ii) The two cards drawn one after other with replacement.</td><td>[L5][CO5]</td><td>[6M]</td></tr>
<tr><td></td><td>b)</td><td>Determine (i) $P\left({}^{B}/_{A}\right)$ (ii) $P\left({}^{A}/_{B^c}\right)$ if A and B are events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A \cup B) = \frac{1}{2}$.</td><td>[L5][CO5]</td><td>[6M]</td></tr>
<tr><td>3</td><td></td><td>Two dice are thrown. Let A be the event that the sum of the point on the faces is 9. Let B be the event that at least one number is 6.
Find (i) $P(A\cap B)$ (ii) $P(A\cup B)$ (iii) $P(A^c\cup B^c)$ (iv) $P(A^c\cap B^c)$ (v) $P(A\cap \boldsymbol{B^c})$</td><td>[L1][CO5]</td><td>[12M]</td></tr>
<tr><td>4</td><td></td><td>In a certain college 25% of boys and 10% of girls are studying mathematics. The girls Constitute 60% of the student body.
(a) What is the probability that mathematics is being studied?
(b) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl?
(c) a boy</td><td>[L1][CO5]</td><td>[12M]</td></tr>
<tr><td>5</td><td></td><td>A random variable X has the following probability function
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>P(x)</td><td>0</td><td>K</td><td>2K</td><td>2K</td><td>3K</td><td>K^2</td><td>$2K^2$</td><td>$7K^2+K$</td></tr>
</table>
Determine (i) K (ii) Evaluate $P(X\geq 6)$ and $P(0<X<5)$ (iii) if $P(X\leq K)>1/2$, find the minimum value of K (iv) variance.</td><td>[L5][CO5]</td><td>[12M]</td></tr>
<tr><td>6</td><td>a)</td><td>Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$, $for\ n \geq 2$, given that $a_0 = -1$ and $a_1 = 8$.</td><td>[L3][CO5]</td><td>[6M]</td></tr>
<tr><td></td><td>b)</td><td>Solve $a_n = a_{n-1} + 2a_{n-2}$ with initial conditions $a_0 = 2$, $a_1 = 7$</td><td>[L3][CO5]</td><td>[6M]</td></tr>
<tr><td>7</td><td></td><td>Solve the recurrence relation
$a_n + 4a_{n-1} + 4a_{n-2} = 8\ for\, n \geq 2, and\ a_0 = 1, a_1 = 2.$</td><td>[L3][CO5]</td><td>[6M]</td></tr>
<tr><td>8</td><td>a)</td><td>Solve the recurrence relation
$a_{n+2} + 3a_{n+1} + 2a_n = 3^n\ for\, n \geq 0\, given\ a_0 = 0, a_1 = 1.$</td><td>[L3][CO5]</td><td>[6M]</td></tr>
<tr><td></td><td>b)</td><td>Solve the following $y_{n+2} - y_{n+1} - 2y_n = n^2$</td><td>[L3][CO5]</td><td>[6M]</td></tr>
<tr><td>9</td><td></td><td>Find a generating function for the recurrence relation
$a_{n+2} - 3a_{n+1} + 2a_n = 0,\quad n \geq 0\ and\ a_0 = 1, a_1 = 6$. Hence solve the relation.</td><td>[L1][CO5]</td><td>[12M]</td></tr>
<tr><td>10</td><td>a)</td><td>In a sample of 100 logic chips, 23 have a defect D_1,26 have a defect D_2 , 30 have a defect D_3 ,7 have defects D_1 and D_2 , 8 have defects D_1 and D_3 ,10 have defects D_2 and D_3 and 3 have all the three defects. Find the number of chips having (i) at least one defect,(ii) no defect.</td><td>[L3][CO5]</td><td>[6M]</td></tr>
<tr><td></td><td>b)</td><td>Find how many integers between 1 and 60 that are divisible by 2 not by 3 and not by 5. Also determine the number of integers divisible by 5 not by 2, not by 3.</td><td>[L3][CO5]</td><td>[6M]</td></tr>
</table>

UNIT –V

GRAPHS

1	**a)**	Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.	**[L2][CO6]**	**[6M]**
	b)	How many vertices will the graph contains 6 edges and all vertices of degree 3	**[L2][CO6]**	**[6M]**
2	**a)**	How many edges does a graph have if it has vertices of degree 4, 3, 3, 2, 2? Draw such a graph.	**[L2][CO6]**	**[6M]**
	b)	If G is non-directed graph with 12 edges, Suppose that G has 6 vertices of degree 3 and the rest have degree less than 3. Determine the minimum number of vertices.	**[L3][CO6]**	**[6M]**
3	**a)**	Explain about complete graph and Bipartite graph with an example.	**[L2][CO6]**	**[6M]**
	b)	Define (i) Planar and non-planar graph (ii) Regular graph with example.	**[L1][CO6]**	**[6M]**
4	**a)**	Define the following graph with one suitable example for each graphs (i) sub graph (ii) induced sub graph (iii) spanning sub graph	**[L1][CO6]**	**[6M]**
	b)	Explain graph coloring and chromatic number give an example.	**[L2][CO6]**	**[6M]**
5	**a)**	Define (i) Isomorphic graph (ii) Multiple graph with example.	**[L1][CO6]**	**[6M]**
	b)	Let G be a 4 – Regular connected planar graph having 16 edges. Find the number of regions of G.	**[L3][CO6]**	**[6M]**
6	**a)**	Show that the two graphs shown in figure are isomorphic?  	**[L2][CO6]**	**[6M]**
	b)	Show that in any graph the number of odd degree vertices is even .	**[L2][CO6]**	**[6M]**
7	**a)**	Write difference between Hamiltonian graphs and Euler graphs.	**[L1][CO6]**	**[6M]**
	b)	Draw the graph represented by given Adjacency matrix (i) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$	**[L1][CO6]**	**[6M]**
8	**a)**	Show that the two graphs shown below are isomorphic? 	**[L2][CO6]**	**[6M]**
	b)	A connected planar graph has 10 vertices each of degree 3. Find the number of regions	**[L3][CO6]**	**[6M]**
9	**a)**	Explain indegree and out degree of a graph. Also explain about the adjacency matrix representation of graphs. Illustrate with an example?	**[L2][CO6]**	**[6M]**
	b)	Suppose a graph has vertices of degree 0 , 2, 2, 3 and 9 . How many edges does the graph have ?	**[L2][CO6]**	**[6M]**
10	**a)**	State Euler's formula and Handshaking theorem	**[L1][CO6]**	**[6M]**

	b)	Find the number of vertices, number of edges and the number of regions for the following graph and verify the Euler's formula 	**[L3][CO6]**	**[6M]**

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